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A METHOD FOR ANALYSING THE EFFECT OF FLOW ON HEAT TRANSFER IN DIE CASTING

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ABSTRACT

A new method has been developed for determining the effect of flow on heat transfer in die casting during steady production. Aside from the assumption that the casting is thin in comparison with its overall size, there are no restrictions on either the die or casting geometry. Important questions concerning the nature and significance of heat transfer during flow are addressed by analysing a representative two-dimensional die. The results show that the effect of flow can be critical when the fill time is long, the casting is thin, or the thermal conductance of the die lubricant is high.

KEY WORDS Die casting Flow effect

INTRODUCTION

A die casting die consists of two or more components that form a cavity when closed. To regulate heat removal from the casting, a coolant, such as water, is usually circulated through conduits drilled in one or more die components. The cycle begins when hot liquid metal is injected into the cavity. Some time after the metal cools and solidifies, the dies are opened and the casting is ejected. After the exposed cavity surfaces have been sprayed with fresh lubricant, the die is closed and the cycle repeats. Eventually, the start-up transients decay and the die temperatures become periodic. If the die is relatively large, it can take one or more hours to progress from the start-up phase to steady operating conditions.

Proper control of die temperatures is a major concern in ensuring quality die castings[1,2]. This is evidenced by the common practice of installing elaborate cooling systems to maintain die temperatures at appropriate levels. Casting defects due to improper cooling include shrinkage porosity, incomplete fill and loss of dimensional control. Despite the importance of cooling design, the development of effective thermal analysis techniques for die casting has been slow. One reason is that related computational research has been oriented towards developing traditional finite element/difference methods for analysing single cycle processes such as sand casting[3]. The situation in die casting is considerably more complex because the steel dies retain the thermal imprint from previous cycles. Although the usual transient finite element/difference

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schemes are applicable, numerical integration costs incurred while tracking the response from start-up to the steady periodic operating conditions can be excessive.

Most applications of the aforementioned thermal analysis techniques in die casting rely on the so-called instant fill assumption. Ordinarily, this assumption is justified by the fact that the fill times encountered in conventional die casting tend to be extremely short. Nevertheless, the effect of flow on heat transfer is likely to become increasingly important due to the current interest in utilizing lower injection rate processes to produce higher quality castings.

Although it is possible to account for the effect of flow using the previously noted computational techniques, one must contend with a substantially reduced time scale. This can translate into a prohibitive increase in computational cost, especially when dealing with complex three-dimensional die models[4,5]. One way to lower the costs of standard solution procedures is to ignore periodicity requirements. In such a strategy, one assumes a set of initial die temperatures and integrates the governing equations over a single casting cycle[6,7]. Any computational advantage afforded by this ad hoc procedure is not likely to offset the penalty of reduced accuracy.

An effective alternative, which avoids successive numerical integrations by determining periodic solutions directly, was introduced by Caulk[8] for symmetric problems and extended by Barone and Caulk[1] to account for asymmetric thermal coupling between die components of arbitrary shape. This method is based on the concept of a transient surface layer in the die. For the layer-based methodology to apply, the casting thickness must be small in comparison with its overall size. Several important modelling simplifications emerge when this condition is satisfied and the conductivity of the thin casting is high, relative to that of the die steel. First, the casting can be represented by a surface with an associated areal mass density to represent variations in its thickness. Second, transverse temperature gradients through the casting thickness can be neglected. The additional observation that the cycle time is usually much shorter than the start-up transient implies that the characteristically large swings in die temperatures, caused by the periodic injection of hot metal, are confined to a relatively thin layer near the cavity surfaces.

Despite the previously noted advantages of the layer method[1,8], it is still limited to problems where the instant fill assumption applies. Nevertheless, the computational appeal of this approach, coupled with a growing need for including the effect of flow on heat transfer in die casting, is sufficient reason for considering an extension. The present paper features a novel generalization that overcomes the instant fill limitation while retaining the essential computational advantages inherent in the original layer-based methodology. Critical additions to the layer formulation[1] include the use of spatially-varying initial casting temperatures and impulsive heat fluxes to account for heat that is lost by the liquid metal and absorbed by the die during flow. Both of these quantities must be obtained from a separate solution of the energy equation during fill. To focus attention on the primary analytical contribution, we develop and solve the energy equation assuming that the velocity field during flow is known. Moreover, convective terms are represented in terms of thickness averaged velocities. Justification for this latter assumption stems from the fact that the castings are thin in comparison with their overall sizes. Since fill times, even for slow flow, are short relative to cycle times, the depth to which transient temperatures penetrate into the die during flow is correspondingly small. This implies that most of the die can be ignored when analysing heat conduction during flow.

In general, the amount of heat transferred during flow depends on the die temperatures just before injection. Die temperatures, in turn, are a direct function of the heat transferred during flow. The fact that the explicit form of this dependence is encumbered by an extremely complex spatial interaction, as well as a material non-linearity due to the presence of latent heat, makes it necessary to employ an iterative solution method. The iteration is initiated by solving for die temperatures using the instant fill assumption. After analysing the problem of heat transfer during fill, the generalized layer analysis is then used to determine a corrected set of die temperatures which form the new starting values for the next iteration.

One iteration is usually sufficient for practical convergence of the generalized layer approach. Assuming a flow solution is available and essentially independent of heat transfer, the computational investment in applying the present approach entails little more than two steady heat conduction analyses of the die and one transient heat conduction analysis of the liquid metal during flow.

The analytical basis for the previously noted computational advantages will become apparent in the formal development to follow. First, we derive the energy equations valid during flow and discuss their role in the overall solution process. Further examination of these equations yields an important non-dimensional number which indicates how key process parameters affect die temperatures. Next, we extend the layer equations in Reference 1 to include the effect of flow and use the resulting generalized contact surface conditions to formulate the iterative solution method. Lastly, we consider results for a representative two-dimensional die to verify convergence and determine when flow is significant.

HEAT TRANSFER DURING FLOW

As a first step in the analytical development, we consider flow as it relates to heat transfer in die casting. The assumption that the transverse thermal resistance of the liquid metal is negligible in comparison with that of the die lubricant implies that thickness variations in the casting temperatures, θ_c, can be neglected. Since our overriding concern is with the thermal impact of convective energy transport during fill and not with details of the actual flow field, the liquid metal velocities are averaged across the cavity thickness, d_c. Consequently, we define a surface representation of the casting in terms of the curvilinear coordinates x_α ($\alpha = 1, 2$), such that $\theta_c = \theta_c(x_\alpha, t)$ and $\mathrm{V} = \mathrm{V}(x_\alpha, t)$, where V is the thickness-averaged velocity. In this case, the time t is reckoned from the onset of injection. The same curvilinear coordinates, x_α, together with an inwardly directed normal coordinate z ($z = 0$ on the cavity surfaces) locate nearby points in adjacent die components (*Figure 1*). The corresponding temperature fields in the die are designated by $\theta^\beta(x_\alpha, z, t)$, where $\beta = 1, 2$.

In deriving the appropriate energy equations we exploit the fact that heat conduction tangential to the cavity wall in both the casting and adjacent regions of the die is negligible during cavity fill. Finally, we ignore any casting temperature change due to viscous dissipation. If $\theta_s^\beta = \theta_s^\beta(x_\alpha, 0, t)$ and h_β denote the temperatures on the two adjacent cavity surfaces and the corresponding thermal conductance of the die lubricant, respectively, then the appropriate energy equation for the liquid metal is:

$$\rho_c c_c d_c \frac{D\theta_c}{Dt} = -\sum_{\beta=1}^{2} h_\beta(\theta_c - \theta_s^\beta) \tag{1}$$

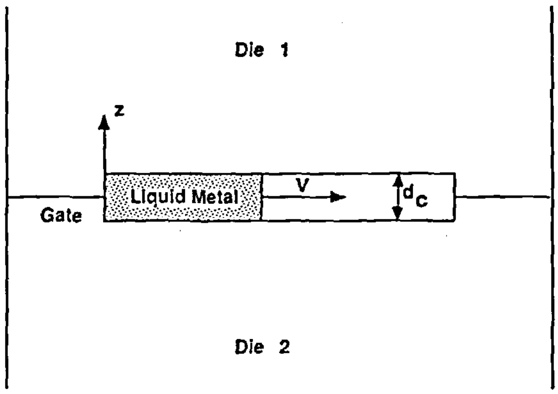

Figure 1 Partially filled die cavity

where

$$\frac{D\theta_c}{Dt} = \frac{\partial\theta_c}{\partial t} + \mathbf{V}\cdot\nabla\theta_c \tag{2}$$

Note that ρ_c and c_c refer to the mass density and specific heat of the liquid metal, respectively. To account for the possible release of latent heat during flow we allow c_c to vary with temperature. Consequently, (1) is materially non-linear.

The presence of the last term in (1), which accounts for thermal coupling between the liquid metal and cavity surface via the conductance of the die lubricant, dictates the need for additional equations governing heat flow into the adjacent die components. Since the dominant direction of heat conduction during flow is normal to the cavity surface, these equations reduce to:

$$\frac{\partial\theta^\beta}{\partial t} = \alpha_\beta \frac{\partial^2\theta^\beta}{\partial z^2} \tag{3}$$

where α_β is the thermal diffusivity associated with adjacent die components. Equation (3), together with (1) and the appropriate initial/boundary conditions enable us to solve for the casting temperatures and heat flux into the die during fill.

Equations (1) and (3) can be written in a more physically revealing form by non-dimensionalizing with respect to the cavity length, L_c, the fill time, t_f, and the injection temperature, θ_{inj}. After making the appropriate substitutions in (1) we obtain:

$$\frac{D\Theta_c}{Dt'} = -\frac{ht_f}{\rho_c c_c \Delta_c}(\Theta_c - \hat{\Theta}_s) \tag{4}$$

where $\Delta_c = d_c/2$, $h = (h_1 + h_2)/2$ and $\hat{\Theta}_s = (h_1\Theta_s^1 + h_2\Theta_s^2)/2h$.

If we define a dimensionless length z' into the die in terms of a thermal penetration depth δ, the corresponding equation for the adjacent die components is:

$$\frac{\partial\Theta^\beta}{\partial t'} = \frac{\alpha_\beta t_f}{\delta^2}\frac{\partial^2\Theta^\beta}{\partial z'^2} \tag{5}$$

The requirement that the heat flux be continuous at the casting–die interface ($z' = 0$) can be expressed as:

$$-\frac{\partial\Theta^\beta}{\partial z'} = \frac{h_\beta\delta}{k_\beta}(\Theta_c^\beta - \Theta_s^\beta) \tag{6}$$

where k_β denotes thermal conductivity of the die steel. Lastly, the respective boundary and initial conditions at the gate ($x_\alpha = x_\alpha^{(g)}$) and in the die are:

$$\Theta_c(x_\alpha^{(g)}, t) = 1 \tag{7}$$

$$\Theta_s^\beta(x_\alpha, z, 0) = \theta^\beta(x_\alpha, z, 0)/\theta_{inj} \tag{8}$$

Note that the term $\theta^\beta(x_\alpha, z, 0)$ refers to the pre-injection temperatures in adjacent die regions $0 \leqslant z \leqslant \delta$, where $\delta \propto \max\sqrt{\alpha_\beta t_f}$.

A qualitative analysis of (4) leads to a better understanding of the significance of energy transport during flow and its effect on die temperatures. We remark that the factor on the right hand side of (4):

$$\frac{ht_f}{\rho_c c_c \Delta_c} \tag{9}$$

can be viewed as a dimensionless heat transfer coefficient relating heat transfer through the cavity walls to convective heat transfer in the axial direction of the cavity. Hence, for a given die design, as the magnitude of this dimensionless number increases we expect to see an exacerbation of the effect of flow on the thermal response of the die. In particular, the influence of flow will increase anytime one or more of the following occurs:

(1) an increase in the conductance of the die lubricant or coating;
(2) an increase in fill time;
(3) a reduction in casting density or specific heat;
(4) a reduction in casting thickness.

The effect of thickness is best understood by recognizing that Δ_c is analogous to a hydraulic diameter and represents the ratio of the cross-sectional area available for convective energy transport to the cavity perimeter available for heat transfer to the die. Lastly, it is important to point out that the cavity length does not explicitly appear in (9). Additional results justifying the absence of this variable will be provided later in the specific context of an example problem.

Obviously, any attempt to incorporate the effect of flow in some type of predictive capability would be complicated by the widely disparate time scales associated with cavity fill and the casting cycle. Typically, fill times for large aluminium castings range from 0.1 to 0.5 sec in contrast to cycle times of approximately one minute and start-up transients of several hours. As indicated previously, the extension of conventional schemes to include the effect of flow is conceptually straightforward but the extremely small time steps required during the flow make numerical integrations even more expensive than in cases where the instant fill assumption is made. Again, it is our intent to avoid this dilemma by capitalizing on the layer approach outlined earlier. In particular, we benefit from the fact that numerical difficulties that usually arise when there is a large disparity between cycle and start-up times have already been overcome. Hence, one need only contend with the new issue of short but finite fill times. An appropriate treatment for this problem will be presented in the next section.

BASIC FORMULATION AND APPROACH

Transient surface layer

Since the present formulation generalizes the transient surface layer approach in Reference 1, we start by reviewing elements of the analysis which are common to both. In general, the layer depth d can be expressed as[8]:

$$d = 1.5(\alpha t_c)^{1/2} \tag{10}$$

where α and t_c denote the thermal diffusivity of the die material and cycle time, respectively. Again, we locate points in the die that are near the casting in terms of the two curvilinear coordinates, x_α, defined on the cavity surface, and the corresponding inwardly directed normal coordinate z. In this case, however, $0 < z < d$. The temperature in the layer, $\theta(x_\alpha, z, t)$, is approximated by the method of weighted residuals based on an N-th order polynomial. More specifically, if we let $\theta_0(x_\alpha)$ denote the interface temperature between the layer and the steady interior region of the die, then:

$$\theta(x_\alpha, z, t) = \theta_0(x_\alpha) + \sum_{m=1}^{N} \theta_m(x_\alpha, t)(1 - z/d)^m \qquad 0 \leqslant z \leqslant d \tag{11}$$

where the N time-dependent coefficients, $\theta_m(x_\alpha, t)$, represent generalized temperatures. Since heat conduction in the x_α direction is neglected, the corresponding energy equation in each layer can be approximated by minimizing the integral of the residual, weighted by the trial functions in

(11). It follows that the desired integral is simply:

$$\int_0^d \left(\rho c \frac{\partial \theta}{\partial t} + \frac{\partial q}{\partial z}\right)(1 - z/d)^n \, dz = 0 \qquad (n = 0, 1, \ldots, N) \tag{12}$$

where q represents the heat flux associated with $\theta(x_\alpha, z, t)$. Note that the constants ρ and c refer to the density and heat capacity of the die steel, respectively. After carrying out the indicated operations for both adjacent layers ($\beta = 1, 2$) we have:

$$\rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+1} \dot{\theta}_m^\beta + q_i^\beta = q_s^\beta \tag{13}$$

$$\rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+n+1} \dot{\theta}_m^\beta + (k_\beta/d_\beta) \sum_{m=1}^{N} \frac{mn}{m+n-1} \theta_m^\beta = q_s^\beta \tag{14}$$

$$(\beta = 1, 2;\ n = 1, 2, \ldots, N)$$

where k_β denotes thermal conductivity and q_s^β the heat flux applied at the surface of each layer. Equation (13) is merely an ancillary condition for determining the flux q_i^β at the interface once (14) has been solved for the generalized temperatures θ_m^β.

A cursory examination of the capacitance terms in (14) may give rise to questions concerning numerical accuracy and robustness. Strictly speaking, the capacitance matrix associated with (14) is inherently ill conditioned and in fact is singular in the limit when $N \to \infty$. Nevertheless, previous analyses[1,8] as well as our results indicate that remarkably accurate solutions can be obtained using a relatively small number of terms in (11). Since the aforementioned studies feature broadly representative examples, adverse conditioning is unlikely to be a factor in any of the intended applications.

It follows from (14) that the solution for θ_m^β depends entirely on the flux terms q_s^β associated with adjacent die surfaces. However, before defining appropriate expressions for these quantities we must first consider the energy balance in the casting. Since it is assumed that the temperature $\theta_c(x_\alpha, t)$ is uniform across the casting thickness, d_c, the energy equation can be expressed as:

$$-\mu c_c \dot{\theta}_c = q_s^1 + q_s^2 \quad \text{for} \quad 0 \leqslant t \leqslant t_r \tag{15}$$

where $\mu = \rho_c d_c$, c_c and t_r represent the areal mass density, specific heat and residence time of the casting, respectively. In general, the specific heat of the casting is a non-linear function of the absolute temperature. Here, it suffices to employ the following stepwise representation:

$$c_c = \begin{cases} c_0 & \theta_c > \theta_{\text{liq}} \\ c_0 + Q_L/(\theta_{\text{liq}} - \theta_{\text{sol}}) & \theta_{\text{liq}} > \theta_c > \theta_{\text{sol}} \\ c_0 & \theta_{\text{sol}} > \theta_c \end{cases} \tag{16}$$

defined in terms of the liquidus and solidus temperatures θ_{liq} and θ_{sol} and the latent heat per unit mass of the casting, Q_L.

The above development is essentially the same as in Reference 1. Subsequently, we address new issues that pertain to the forthcoming generalization for flow. In this case, the desired expressions for q_s^β should reflect the thermal interaction between the casting and adjacent die surfaces, both during and after flow. First, consider the injection phase of the casting cycle and let $Q_f^\beta(x_\alpha)$ denote the heat per unit area absorbed by the die at x_α during the fill time t_f. Since the fill time t_f is generally much less than the cycle time t_c, the energy exchange during flow can be regarded as taking place instantly at the start of the cycle when $t = 0$. In effect, the die is subject to an impulsive heat flux defined as:

$$q_s^\beta = Q_f^\beta \delta(t) \tag{17}$$

where $\delta(t)$ denotes the Dirac delta function. If (17) is incorporated in the analysis of (14), conservation of energy requires that we also impose the residual temperature of the liquid metal just after fill as the initial condition at $t = 0^+$. In other words,

$$\theta_c^0 = \theta_c(x_\alpha, 0^+) \tag{18}$$

Obviously, (18) represents a significant departure from the previous layer analysis in Reference 1, which neglects the effect of flow and assumes a uniform initial temperature equal to the injection temperature θ_{inj}. Note that the residual temperature θ_c^0 as well as Q_f^β are computed quantities resulting from the analysis of heat transfer during flow discussed earlier.

After 'flow', when $0 < t < t_c$, the relationships that define q_s^α are the same as in Reference 1. Since the contact resistance due to the die lubricant during and after the residence time t_r is finite we let:

$$q_s^\beta = \begin{cases} h_\beta(\theta_c - \theta_s^\beta) & 0 < t < t_r \\ h_0(\theta_a - \theta_s^\beta) & t_r \leqslant t < t_c \end{cases} \quad \text{on } \partial C \tag{19}$$

where h_β and h_0 represent the conductance of the lubricant on the cavity surface, ∂C, before and after ejection. In this case, θ_a denotes the ambient temperature.

Jump conditions

One of the unique features of the die casting process is that die temperatures are subject to periodicity conditions, while initial conditions are imposed on the casting. In short, the solution must satisfy a coupled periodic/initial value problem. As indicated in (18), the proper initial condition for the casting is the corresponding residual temperature just after cavity fill. If it were not for the impulsive flux the appropriate periodicity condition for the die temperatures would be the same as in Reference 1. In the present case, however, some additional development is required. After substituting the impulsive flux, (17), into the basic layer equations, (14), we integrate to obtain:

$$\int_{-\tau}^{\tau} \left\{ \rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+n+1} \dot{\theta}_m^\beta + (k_\beta/d_\beta) \sum_{m=1}^{N} \frac{mn}{m+n-1} \theta_m^\beta \right\} \mathrm{d}\tau = \int_{-\tau}^{\tau} \{ Q_f^\beta \delta(t) + q_s^\beta \} \, \mathrm{d}\tau \tag{20}$$

where q_s^β is defined in (19). The limit as $\tau \to 0$ can be evaluated following explicit integration of the first and third terms in (20). Since the remaining integrands are bounded functions, the corresponding integrals vanish as $\tau \to 0$ and the final expression reduces to the jump condition:

$$\rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+n+1} [\![\theta_m^\beta]\!] = Q_f^\beta \qquad (n = 1, 2, \ldots, N) \tag{21}$$

where

$$[\![\theta_m^\beta]\!] = \theta_m^\beta(0^+) - \theta_m^\beta(t_c^-) \tag{22}$$

Note, for the special case of instant fill, $Q_f^\beta = 0$ and (21) reduces to the periodicity condition, $[\![\theta_m^\beta]\!] = 0$, introduced in Reference 1. In the present formulation the entire effect of flow is embodied in (21) and the corresponding initial condition, (18).

The addition of the above jump condition completes the generalization of the layer formulation in Reference 1. We can now solve the governing equations, (14) and (15), for the corresponding generalized temperatures with the help of (16), (18), (19) and (21), provided the temperatures θ_0^β at the interface between the transient surface layer and steady interior region are known. It turns out that interface temperatures can be conveniently extracted from a steady analysis of the die, prior to recovering transient temperatures in the layer. The viability of this approach hinges on the preliminary linearization of (16) introduced by Barone and Caulk[1]. Basically, the idea is to replace the non-linear representation for the casting material (16) by the constant sensible thermal capacitance, c_0. The energy associated with the release of latent heat can then be accounted for by defining a new initial casting temperature, θ_c^*, as follows:

$$\theta_c^* = \theta_c^0(x_\alpha) + \bar{Q}_L(x_\alpha)/c_0 \tag{23}$$

where $\bar{Q}_L(x_\alpha)$ denotes the amount of latent heat left in the casting at x_α when fill is complete. Again, once the steady solution has been determined, we can recover the transient temperatures in the layer by solving the fully non-linear equations with specified interface temperatures. In general, the integration is continued over successive casting cycles until the jump conditions in (21) are satisfied. Fortunately, if the pre-injection temperatures based on the linearized analysis are used as starting conditions, (21) is usually satisfied within a few cycles.

By exploiting the linearized solution of the layer equations we are able to generate appropriate contact surface conditions that relate the time-averaged fluxes and temperatures on the cavity surface, thereby making it possible to solve the steady interior problem in a reasonably straightforward fashion. Although a detailed analysis, pertaining to instant fill, is available in Reference 1, those elements of the development which support the present geeneralization will be included in the next section for completeness.

Contact surface conditions

As indicated above, the approach for solving the transient layer equations requires that we first determine the steady underlying temperatures in the die. Obviously, the crucial step here is to establish the boundary conditions on the cavity surface that properly account for the casting and any thermal coupling between adjacent die components. These conditions follow directly from the time averaged solutions of the linearized layer equations. In this case, unessential detail is avoided by letting $h_0 = 0$ in (19). After substituting this version of (19) in (14) and (15), we use (23) to obtain the following linearized expressions valid during the residence time ($0 < t < t_r$):

$$\rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+n+1} \dot{\theta}_m^\beta + (k_\beta/d_\beta) \sum_{m=1}^{N} \frac{mn}{m+n-1} \theta_m^\beta = h_\beta\left(\theta_c^\beta - \sum_{m=1}^{N} \theta_m^\beta\right) \tag{24}$$

and

$$\mu c_0 \dot{\theta}_c^\beta = \sum_{\beta=1}^{2} h_\beta\left(\theta_c^\beta - \sum_{m=1}^{N} \theta_m^\beta\right) \tag{25}$$

where

$$\theta_c^\beta = \theta_c - \theta_0^\beta \tag{26}$$

A similar procedure applied to the post ejection period when ($t_r \leqslant t < t_c$) yields

$$\rho_\beta c_\beta d_\beta \sum_{m=1}^{N} \frac{1}{m+n+1} \dot{\theta}_m^\beta + (k_\beta/d_\beta) \sum_{m=1}^{N} \frac{mn}{m+n-1} \theta_m^\beta = 0 \tag{27}$$

Note that the change of variables defined in (26) is used to transform the governing equations to the indicated homogeneous form as in Reference 1. Clearly, the price for this simplification is that there are now two casting equations $(25)_{1,2}$ subject to the following initial conditions:

$$\theta_c^\beta(x_\alpha, 0^+) = \theta_c^* - \theta_0^\beta \tag{28}$$

In addition to the above initial conditions, the linear solution must also satisfy the jump conditions (21). Prior to enforcing these conditions, the two sets of equations valid for the residence and post ejection times are each expressed as a linear combination of their respective eigenfunctions. The redundant coefficients are eliminated by applying temperature continuity at $t = t_r$. Ultimately, the solution to the resulting set of equations is used to determine an expression for time averaged temperatures on the cavity surface, $\bar{\theta}_s^\beta$. To expedite the forthcoming development we first recall[8] that:

$$\bar{\theta}_s^\beta = \theta_0^\beta + \sum_{m=1}^{N} \bar{\theta}_m^\beta = \theta_0^\beta + \bar{\theta}_1^\beta \tag{29}$$

and then observe, on the basis of (24)–(27), that the solution is linear and homogeneous in the initial temperature $\theta_c^* - \theta_0^\beta$ and the heat absorbed during flow Q_f^β. Hence, we can express $\bar{\theta}_1^\beta$ as:

$$\bar{\theta}_1^\beta = \sum_{\gamma=1}^{2} [A_{\beta\gamma}(\theta_c^* - \theta_0^\gamma) + B_{\beta\gamma}Q_f^\gamma] \tag{30}$$

Each of the coefficients in (30) represents an influence function generated by time averaging transient solutions that satisfy one of four independent unit initial/jump conditions with the remaining three set to zero. If (29) is used to eliminate θ_0^β from (30) then we have:

$$\bar{\theta}_1^\beta = \sum_{\gamma=1}^{2} [a_{\beta\gamma}(\theta_c^* - \bar{\theta}_s^\gamma) + b_{\beta\gamma}Q_f^\gamma] \tag{31}$$

where

$$\begin{aligned} a_{\beta\gamma} &= (\delta_{\beta\sigma} - A_{\beta\sigma})^{-1}A_{\sigma\gamma} \\ b_{\beta\gamma} &= (\delta_{\beta\sigma} - A_{\beta\sigma})^{-1}B_{\sigma\gamma} \end{aligned} \tag{32}$$

and $\delta_{\beta\sigma}$ denotes the Kronecker delta. Since $\bar{q}_s^\beta = (k_\beta/d_\beta)\bar{\theta}_1^\beta$ (see Reference 8), (31) can be expressed as:

$$\bar{q}_s^\beta = (k_\beta/d_\beta) \sum_{\gamma=1}^{2} [a_{\beta\gamma}(\theta_c^* - \bar{\theta}_s^\gamma) + b_{\beta\gamma}Q_f^\gamma] \tag{33}$$

With the help of:

$$b_\beta = \sum_{\gamma=1}^{2} b_{\beta\gamma}Q_f^\gamma \tag{34}$$

(33) can be rearranged more compactly as follows:

$$\begin{aligned} \bar{q}_s^1 &= (k_1/d_1)[(a_{11} + a_{12})(\theta_{cf}^1 - \bar{\theta}_s^1) + a_{12}(\bar{\theta}_s^1 - \bar{\theta}_s^2)] \\ \bar{q}_s^2 &= (k_2/d_2)[(a_{22} + a_{21})(\theta_{cf}^2 - \bar{\theta}_s^2) + a_{21}(\bar{\theta}_s^2 - \bar{\theta}_s^1)] \end{aligned} \tag{35}$$

where

$$\begin{aligned} \theta_{cf}^1 &= \theta_c^* + b_1/(a_{11} + a_{12}) \\ \theta_{cf}^2 &= \theta_c^* + b_2/(a_{22} + a_{21}) \end{aligned} \tag{36}$$

The above equations are the desired contact surface conditions which represent the steady flux on the cavity surface. Not only does (35) have the same form as its counterpart for the case of instant fill given in Reference 1, but the entire effect of flow is captured in the non-uniform distribution of the generalized initial casting temperatures θ_{cf}^1 and θ_{cf}^2.

Since the boundary conditions on exterior die surfaces are unaffected by the casting, they can be expressed in terms of the standard relationship, $(19)_2$. Of course, $(19)_2$ does not apply to parting surfaces, because the amount of heat flow depends on how long the die is closed. In this case, it is necessary to employ a special adaptation of the transient layer analysis developed in Reference 1. Once the boundary conditions on all die surfaces (including cooling lines) have been prescribed, steady die temperatures can be calculated using the special boundary element approach discussed in Reference 1.

Solution method

The basic layer equations and associated contact surface conditions needed in determining underlying steady temperatures in the die were established in the two previous subsections. During the aforementioned development, it was tacitly assumed that the residual temperature $\theta_c^0(x_\alpha)$ and the heat absorbed by the surface layers at the end of fill $Q_f^\beta(x_\alpha)$ are known quantities. Although θ_c^0 and Q_f^β are not known *a priori*, they can be determined from an analysis of heat transfer during flow, provided the initial die temperatures $\theta^\beta(x_\alpha, z, 0)$ prior to injection are available. However, these temperatures are also unknown because they depend on θ_c^0 and Q_f^β. It is also apparent from the previous development that the explicit form of the implied relationship between θ_c^0, Q_f^β and θ^β is encumbered by an externally complex spatial coupling as well as a material non-linearity due to the presence of latent heat. For these reasons we opt for the following iterative solution procedure:

(1) solve for the steady die temperatures assuming instant fill, with $\theta_c^0 = \theta_{\text{inj}} =$ constant and $Q_f^\beta = 0$;
(2) recover the pre-injection temperatures by solving the non-linear transient equations, (14) and (15);
(3) perform the local analysis of heat transfer during flow; this entails solving (4) and (5) to determine the residual casting temp. $\theta_c^0(x_\alpha)$, and the strength of the impulsive flux, $Q_f^\beta(x_\alpha)$;
(4) solve for the steady die temperatures using the generalized contact surface condition (35);
(5) recover the new pre-injection temperatures from (14) and (15);
(6) return to step (3) and repeat the process until the die temperatures converge. We recommend a relative convergence criterion based on a distributed least squared difference in average die temperatures as outlined in the results section.

The above procedure can be applied to any die casting die used to make shell-like parts, provided a suitable capability for flow analysis is available.

Although a general treatment of cavity fill is beyond the scope of the present paper, the two-dimensional example considered in the next section provides a suitable test of convergence. Since the geometry and process conditions considered are representative of actual die casting operations the results also offer some valuable physical insight regarding the effect of flow.

TWO-DIMENSIONAL EXAMPLE

Physical description

We confine attention to aluminium castings and consider the representative two-dimensional die shown in *Figure 2a*. The 0.5 m long die cavity, with a nominal thickness of 6 mm, is centred between two identical die components each 1 m long by 0.5 m deep. Each die component contains two cooling lines spaced 25 cm apart at a depth of 10 cm away from the cavity surface. These die and casting dimensions are indicative of large diecast automotive components. Specific material properties for the die steel and aluminium considered are given in *Table 1* together with fixed process variables. Note that the selected cycle and residence times of 80 and 30 sec, respectively, are representative of the intended applications.

Numerical implementation

The time-averaged die temperatures are computed on the basis of the boundary element model in *Figure 2b*, assuming a uniform distribution of temperature and heat flux on each element. Unessential detail is avoided by imposing adiabatic boundary conditions on all exterior and parting surfaces. Since both the boundary conditions and the geometry are symmetric about the casting centreline only one half of the die is modelled. An additional consequence of the noted symmetry is that the cross-coupling coefficients in the contact surface conditions (35) vanish.

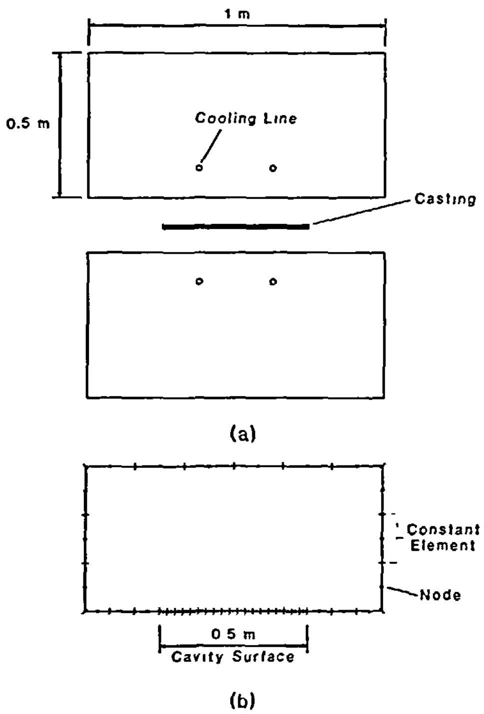

Figure 2 (a) Symmetric two-dimensional die; (b) boundary element model

Table I

Casting properties	
Specific heat, c_c	1000 J/kg K
Density, ρ_c	2570 kg/m^3
Conductivity, k_c	108 W/m K
Latent heat, Q_L	390000 J/kg
Liquidus, θ_{liq}	593°C
Solidus, θ_{sol}	538°C
Die properties	
Specific heat, c	584 J/kg K
Density, ρ	7760 kg/m^3
Conductivity, k	29 W/m K
Process parameters	
Injection temperature, θ_c^0	620°C
Cycle time, t_c	80 sec
Residence time, t_r	30 sec
Water temperature, θ_w	25°C

As indicated earlier, the determination of transient temperatures from the non-linear surface layer equations, (14) and (15), is an essential complement to the underlying steady analysis discussed in the previous paragraph. Although the basic approach used in solving these equations is outlined above, a more complete treatment of the transient recovery procedure as well as the corresponding analysis for steady die temperatures can be found in Reference 1.

For the application considered the layer depth, d_c, calculated from (10) is 34 mm. Additionally, the associated die temperature is expressed as a sixth order polynomial. Previously, Caulk[8] showed that a fourth order polynomial is normally adequate, but did not anticipate the imposition of an impulsive heat flux. The decision to use a sixth order polynomial was made after analysing the impulse response in a half-space. As discussed in the Appendix, this approximation represented the best compromise between accuracy and efficiency.

Recall from the discussion of the solution method that the local energy equations (4) and (5) must be solved at each iteration in order to predict the distributed heat, Q_f, absorbed by the die during flow and the resulting non-uniform casting temperature, θ_c^0. Once the velocity of the liquid metal is specified, the aforementioned equations are solved using a standard finite difference approximation. In this case, we use upwind differencing to approximate the convective term in (4) and apply the Crank–Nicolson implicit method to integrate (5). The cavity is discretized into 38 equal control volumes, each requiring three time steps to fill. Similarly, a die depth equal to $12\sqrt{\alpha t_f}$, ($\alpha = \alpha_1 = \alpha_2$) is divided into 20 equal segments.

Accuracy of the finite difference predictions for Q_f and θ_c^0 was checked by comparing with results obtained from the Laplace transform method. The finite difference predictions, with c_c for aluminium held constant, were virtually indiscernible from the Laplace transform solutions.

RESULTS

By varying the conductance of the die lubricant, h, and the fill time, t_f, we can represent a variety of casting processes. Typically in conventional die casting the die lubricants at the mould/metal interface are very conductive and the fill times are rapid. On the other hand, in permanent mould casting, die coatings are insulative and fill times are long relative to die casting. More modern processes such as squeeze casting are characterized by high to moderate values for the coating conductance, h, and intermediate fill times.

Slow fill

We first examine a case in which the effect of flow is significant. A 6 mm thick cavity, with a moderate h of 10,000 W/m^2 K, is filled with aluminium in 0.5 sec. The results are shown in *Figure 3.* In the upper left quadrant of the Figure the relative convergence rate is plotted *versus* the iteration number. This relative convergence rate is expressed as the root mean square deviation of the average die surface temperature for the iteration in question with respect to the last or in this case fifth iteration. Further iterations produce no appreciable change in the results. Keep in mind that in the zeroth iteration the thermal analysis is performed using the instant fill assumption and only in subsequent analyses is the correction for the effect of flow made. For the first set of process conditions, practical convergence occurs after one or two iterations. Remarkably, as will be evident in subsequent results, even when the effect of flow is large the solution still converges in just one or two iterations.

We next remark on the 'instant fill' average die temperatures, casting ejection temperatures, and solidification times, which are plotted *versus* the normalized distance along the cavity using broken lines (the origin is the gate location). For this die configuration the 'instant fill' predictions are symmetric about the cavity midpoint. Note also the thermal imprint of the

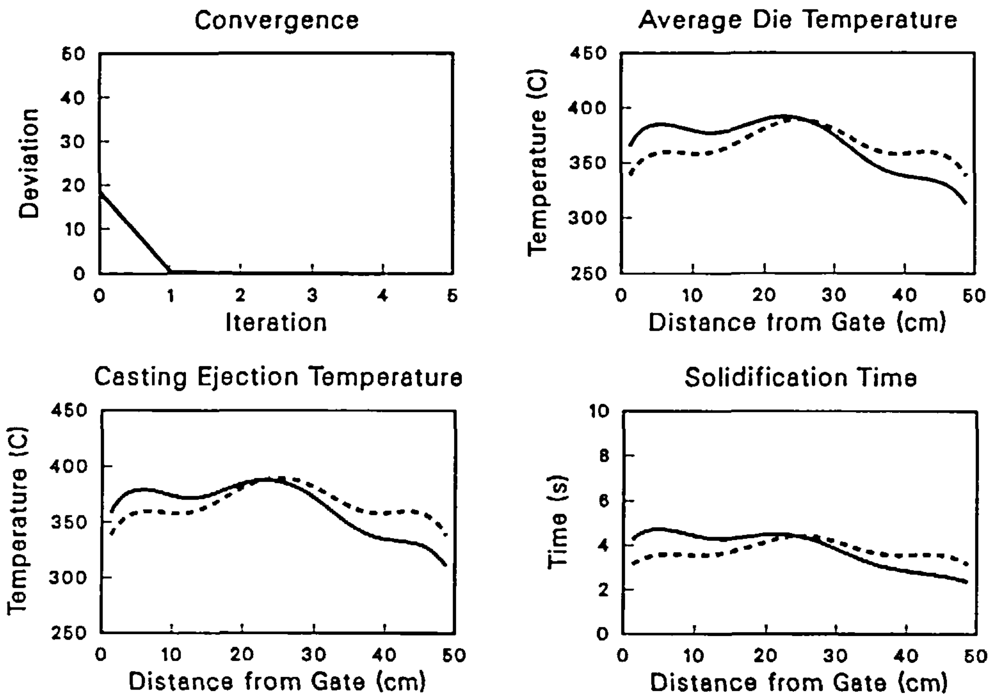

Figure 3 Slow die casting: instant fill (---), finite fill time (———), for h = 10000 W/m^2 K, d_c = 6 mm, and t_f = 0.5 sec

cooling line locations along the cavity as evidenced by the reduced temperatures and solidification times in their vicinity.

Upon introducing the effect of flow we calculate the response indicated by the solid curves in *Figure 3*, superposed on the 'instant fill' results. The net effect of flow is to skew the temperature profiles and solidification times. Near the gate, where the exposure of the die to hot metal is longer than at the end of the cavity, the temperatures are higher and solidification times are longer than with 'instant fill'. At the end of the cavity, where casting temperatures after fill are lower, we see the reverse trend. If we quantify the effect of flow in terms of the temperature difference or skewness between the gate and the end of the cavity, the difference for this casting process is approximately 50°C for average die temperatures as well as casting ejection temperatures. The effect of flow on solidification times (*Figure 3*) naturally follows the same trends as the results for die and casting temperatures. At the gate the liquid metal takes longer to solidify than at the end of the cavity. This makes sense since by the time the liquid metal arrives at the end of the cavity it has already transferred some of its energy to upstream regions of the die. If the solidification time is less than the fill time anywhere in the casting we clearly have a non-viable process since the cavity cannot be completely filled.

In *Figure 4* we show what happens when h for the 6 mm thick cavity is increased to 50000 W/m^2 K. The largest allowable fill time for this set of process parameters is 0.5 sec; at longer fill times solidification occurs before the cavity can be completely filled. As indicated in *Figure 4*, the skewness in die temperatures is about 120°C. Similarly, the predicted ejection temperatures indicate the presence of a large thermal gradient in the casting. This temperature gradient could induce undesirable casting distortion. In spite of these extreme effects practical convergence occurs in only two iterations.

Rapid fill

To determine the effect of flow in a more conventional die casting setting, we model very rapid filling of the cavity with a moderate h of 10000 W/m^2 K. *Figure 5* shows the results for

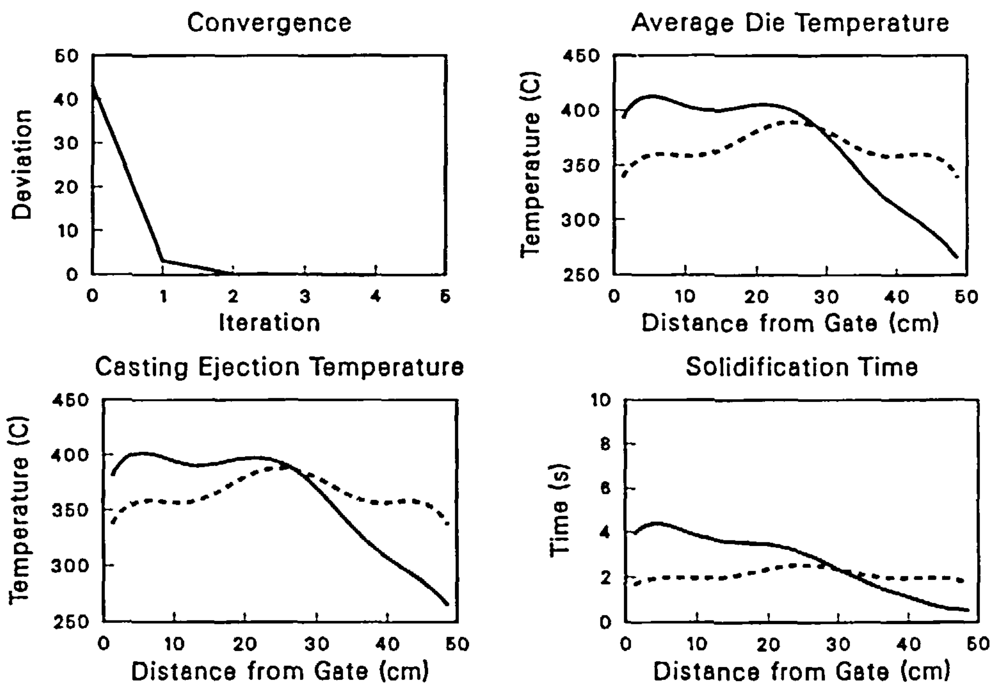

Figure 4 Slow die casting: instant fill (---), finite fill time (———), for h = 50000 W/m^2 K, d_c = 6 mm, and t_f = 0.5 sec

filling the 6 mm thick cavity in 0.1 sec. There is very little difference between the thermal response with and without flow. The mere 10°C difference in average die temperatures with and without flow affirms the instant fill assumption for conventional die casting.

Effect of casting thickness

We summarize the effect of casting thickness in *Figure 6*, where the end-to-end differences in average die temperature are plotted *versus* the fill time for various casting thicknesses ranging from 2 to 10 mm. In this case, h was set at a mid-range value of 30000 W/m^2 K. Note that the skewing of die temperatures increases as castings become thinner. Obviously, thinner walled castings are more difficult to cast without experiencing premature solidification. Notice that the curve corresponding to a 4 mm thick casting is truncated at 0.3 sec. This truncation indicates that at fill times longer than 0.3 sec, the casting solidifies prior to the completion of fill. We see similar results for the 2 mm casting.

Cavity length

Finally, we look at what happens when the overall die dimensions remain constant but the 0.5 m cavity length, cooling line locations, and cooling line diameters are scaled up (down) by a factor of 1.414 (0.707). In *Figure 7* the resulting average die temperatures are shown for a fill time of 0.2 sec and an h value of 30000 W/m^2 K. Changing the overall casting size changes the heat absorbed by the die which in turn alters the underlying steady response in the die. We observe, however, that the magnitude of the end-to-end temperature difference remains virtually unchanged. This is consistent with our earlier dimensional analysis which precluded cavity length as a relevant variable.

Dimensionless number

We have shown that the effect of flow on the overall thermal response in die casting is influenced by a number of parameters including the casting thickness d_c, the fill time t_f and

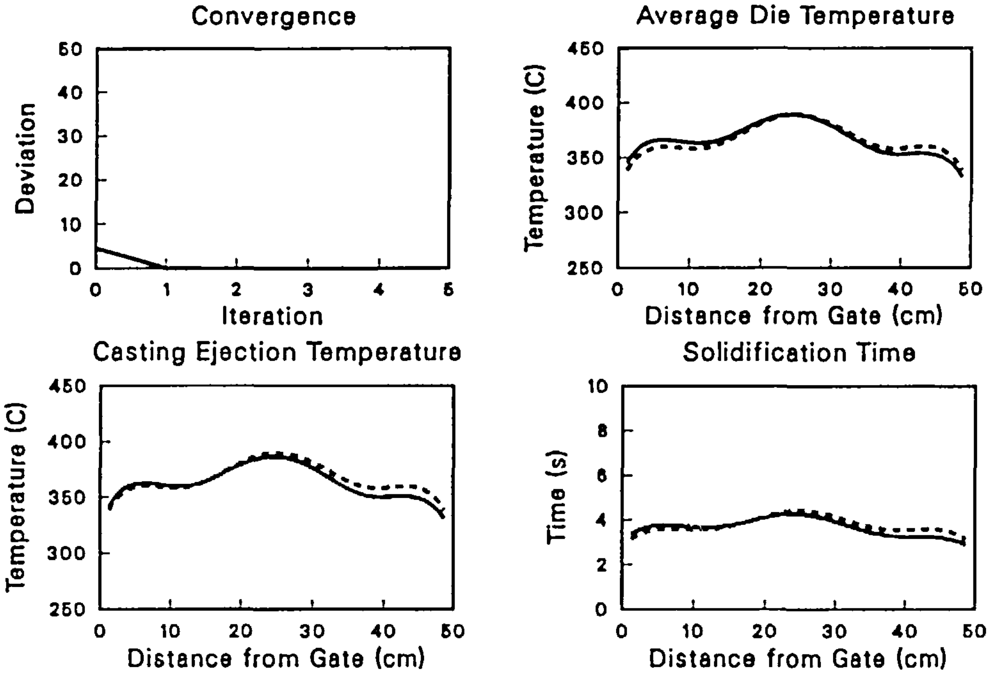

Figure 5 Fast die casting: instant fill (——), finite fill time (———), for h = 10000 W/m^2 K, d_c = 6 mm, and t_f = 0.1 sec

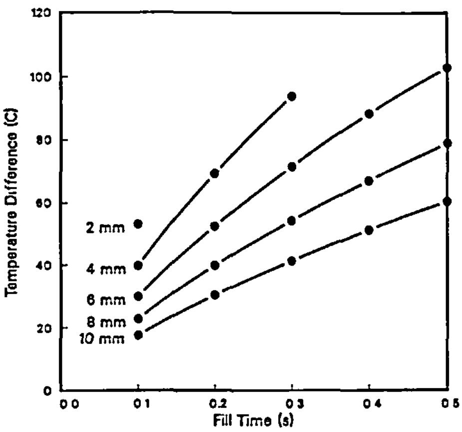

Figure 6 Effect of casting thickness for $h = 30000\ \mathrm{W/m^2\,K}$

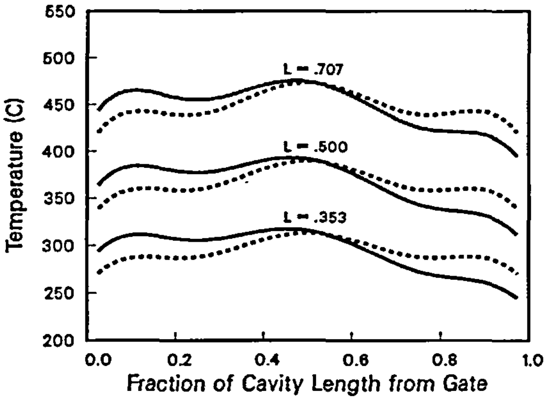

Figure 7 Effect of cavity length on average die temperature: instant fill (----), finite fill time (———), for $h = 30000\ \mathrm{W/m^2\,K}$, $d_c = 6$ mm, and $t_f = 0.2$ sec

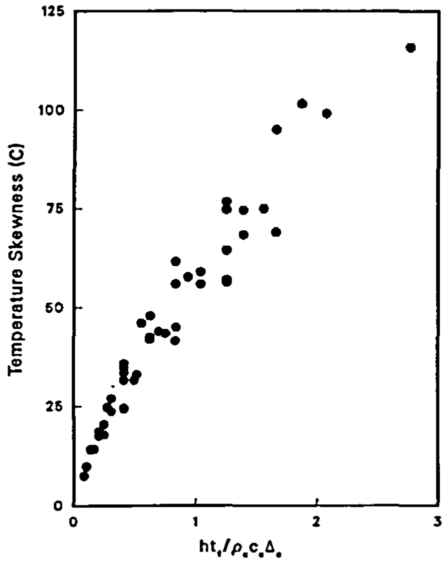

Figure 8 Dimensionless parameter

the coating conductance, h. The relationship between these quantities and the temperature skewness introduced by flow is most conveniently expressed in terms of the dimensionless number in (9). In *Figure 8*, we plot the end-to-end difference in average die temperatures obtained from the layer analysis *versus* (9) for a wide range of process conditions. The corresponding variations in the key parameters are as follows: $0.1 \leqslant t_f \leqslant 5$ sec, $2 \leqslant d_c \leqslant 10$ mm and $1000 \leqslant h \leqslant 50{,}000\ \mathrm{W/m^2\,K}$. Despite some scatter, a clear trend emerges. More importantly, the fact that the relationship is essentially linear confirms the validity of (9) in capturing the primary physical interactions.

ACKNOWLEDGEMENTS

We wish to express our appreciation to David Caulk for sharing valuable analytical insights concerning the basic layer methodology. Contributions by Robert Frutiger made in a prior thermal analysis of one-dimensional cavity fill are also gratefully acknowledged.

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APPENDIX

Although solution errors associated with the layer representation for bounded heat flux boundary conditions were discussed in detail by Caulk[8], the effect of impulsive flux loadings on accuracy is a new issue which requires further clarification. More specifically, we must determine whether the present generalization necessitates an increase in the order of the polynomial representation (11). To this end, we compare layer predictions to an exact solution for the impulse response in a half-space. In *Figure 9* the time history of the normalized surface temperature is plotted on a log scale for polynomials of varying degrees. In *Figure 10* we plot the spatial variation of the internal temperatures half a second after the application of the impulsive heat flux.

In general, we expect to see a rapidly decaying, high amplitude (theoretically infinite) response. The approximation of the response is proportional to the inverse of the capacitance matrix and its decay characteristics relate to the eigenvalues of the corresponding system of first-order differential equations (14). Therefore, as the degree of the polynomial is raised, the form of equation (14) leads us to expect a higher amplitude response to the impulse with a more rapidly

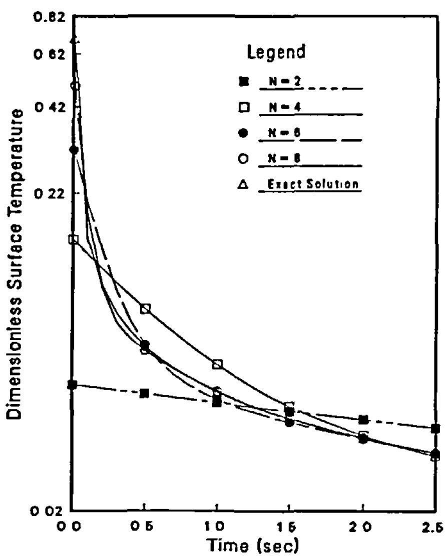

Figure 9 Comparison of exact and approximate surface temperatures on an impulsively heated half-space for successive polynomial orders N in the representation (11)

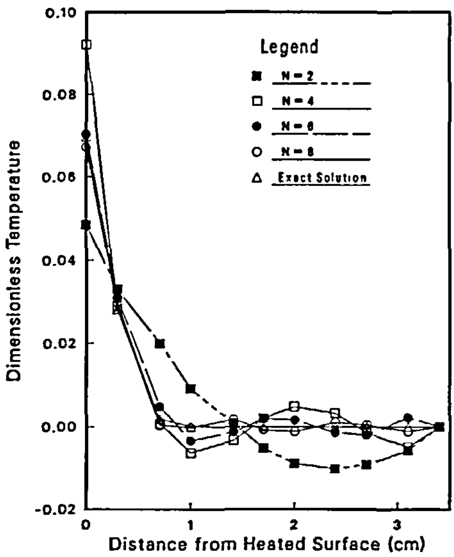

Figure 10 Comparison of exact and approximate temperature distributions in a half space 0.5 sec after the application of an impulsive heat flux for successive polynomial orders *N* in the representation (11)

decaying initial transient. The results indicate that for our purpose, a polynomial of degree six exhibits a sufficiently convergent approximation. The increased accuracy brought by using an eighth-order polynomial is minimal. Additionally, the use of higher order polynomials results in a greater computational cost and reduced robustness.